# MATHEMATICS

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AIM POINTS
MATHEMATICS
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## XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

# STRAIGHT LINE IN SPACE

## & Their Properties

CONTENTS

## THINGS TO REMEMBER

1. Two non-parallel planes always interest in a straight line. Thus, if $a_1$ x $b_1$ y + $c_1$ z + $d_1$ = 0 and $a_2$ x $b_2$ y + $c_2$ z + $d_2$ = 0 are equations of two non-parallel planes, then these two equations taken together represent a line i.e.,

   $a_1 x b_1 y + c_1 z + d_1 = 0 = a_2 x b_2 y + c_2 z + d_2$

   is the equation of a line.

   This is known as an un-symmetrical form of a line.

2. The equation of a line passing through a point ($x_1$, $y_1$, $z_1$) and having direction cosines (or direction ratios), *l*, m, n are given by

   $$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

   The coordinates of a arbitrary point on this line are ($x_1 + lr$, $y_1$ + mr,$z_1$ + nr), where r is a parameter. This is known as symmetrical form of a line.

3. The vector equation of a line passing through a point having position vector $\vec{a}$ and parallel to vector $\vec{b}$ is

   $\vec{r}=\vec{a}+\lambda\vec{b}$, where λ is a parameter.

4. The equations of a line passing through points ($x_1$, $y_1$ $z_1$) and ($x_2$, $y_2$, $z_2$) are given by

   $$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

## EXERCISE-1

1. The cartesian equation of a straight line passing through a fixed point ($x_1$, $y_1$, $z_1$) and having direction ratios proportional to a, b, c is given by

   $$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2. The cartesian equation of a line passing through two given point ($x_1$, $y_1$, $z_1$), ($x_2$, $y_2$, $z_2$) are given by

   $$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

3. Find the direction cosines of the line $\frac{x-2}{2}=\frac{2y-5}{-3}$, z = –1. Also, find the vector equation of the line.

4. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of $3\sqrt{2}$ from the point (1, 2, 3).

5. Find the vector equation for the line which passes through the point (1, 2, 3) and parallel tot he vector $\hat{i}-2\hat{j}+3\hat{k}$. Reduce the corresponding equation in cartesian from.

6. Find the vector equation of a line passing through (2, –1, 1) and parallel to the line whose equations are $\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3}$

7. The cartesian equations of a line are $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

8. Find the direction cosines of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

9. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1, 2, 3).

10. Show that the points whose position vectors are $-2\hat{i}+3\hat{j}, \hat{i}+2\hat{j}+3\hat{k}$ and $7\hat{i}-\hat{k}$ are collinear.

11. Find the certesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$.

12. The cartesian equations of a line are $3x + 1 = 6y - 2 = 1 - z$. Find the fixed point through which it passes, its direction ratios and also its vector equation.

13. Find the equation of the line passing through the point (–1, 3, –2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

14. A line passes through (2, –1, 3) and is perpendicular to the line $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}-2\hat{j}+2\hat{k})$. Obtain its equation.

15. Find the angle between the following pairs of lines :

(i) $\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{3}$ and $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$

(ii) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3}$ and $\frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$

(iii) $\frac{5-x}{-2}=\frac{y+3}{1}=\frac{1-z}{3}$ and $\frac{x}{3}=\frac{1-y}{-2}=\frac{z+5}{-1}$

(iv) $\frac{x-2}{3}=\frac{y+3}{-2}=z=5$ and $\frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$

(v) $\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1}$ and $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$

16. Find the angle between the pairs of lines with direction ratios proportional to
    (i) 5, -12, 13 and -3, 4, 5
    (ii) 2, 2, 1 and 4, 1, 8
    (iii) 1, 2, -2, and -2, 2, 1
    (iv) a, b, c and b - c, c - a, a - b

17. Find the equation of the line passing through the point $\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the lines $\vec{r}=\hat{i}+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(\hat{i}+\hat{j}+\hat{k})$.

18. Determine the equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines

    $$\frac{x-8}{8}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

19. If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, find the value of λ.

20. If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (–4, 3, –6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

21. Find the value of λ so that the following lines are perpendicular to each other.

    $$\frac{x-5}{5\lambda-2}=\frac{2-y}{5}=\frac{1-z}{-1}, \frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

22. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Find their point of intersection.

23. Determine whether the following pair of lines interest or not :

    $$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}, \frac{x-8}{7}=\frac{y-4}{1}=\frac{3-5}{3}$$

24. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

## EXERCISE-2

1. Find the perpendicular distance of the point (3, –1, 11) from the line $\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$.

2. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular.

3. Find the equationof the perpendicular drawn from the point P(–1, 3, 2) tothe line $\vec{r} = (2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+\hat{j}+3\hat{k})$. Also find the coordinates of the foot of the perpendicular from P.

4. Find the foot of the perpendicular from (0, 2, 7) on the line

$$\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$$

5. Find the foot of the perpendicular from (1, 2, –3) to the line

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$$

6. Find the equation of line passing through the point A (0, 6, –9) and B(–3, –6, 3). If D is the foot of perpendicular drawn from a point C(7, 4, –1) on the line AB, then find the coordinates of the point D and the equation of line CD.

7. Find the shortest distance between the line

$$\vec{r} = (4\hat{i}-\hat{j}) + \lambda(\hat{i}+2\hat{j}-3\hat{k})$$

and $\vec{r} = (\hat{i}-\hat{j}+2\hat{k}) + \mu(2\hat{i}+4\hat{j}-5\hat{k})$.

8. Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k})$$

and $\vec{r} = (2\hat{i}+4\hat{j}+5\hat{k}) + \mu(4\hat{i}+6\hat{j}+8\hat{k})$.